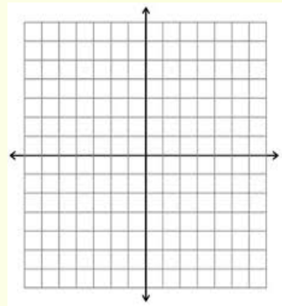
Alg. 2 Warm Up #5-5

1. Graph: $y = \log_2(x-3) - 2$



2. Simplify: $(5x^{-2}y^3)^2(3x^5y^{10})^{-2}$

HW Questions:

6-95. Complete the table at right and find its equation.

x	y
	1
	3
2	9
	27
4	
	243
6	
7	
8	

6-96. Margee thinks she can use logs to solve $\boxed{56 = x^8}$, since logs seem to make exponents disappear. Unfortunately, Margee ~~is wrong.~~ Explain the difference between equations like $2 = 1.04^x$, in which you can use logs, and $56 = x^8$, in which it does not make sense to use logs.

$\sqrt[8]{\ }$

6-97. What values must x have so that $\log(x)$ is greater than 2? Justify your answer.

$\log x > 2$

an exponent on the base 10 > 2

$10^2 = \boxed{100}$ x

$10^{\text{More than } 2}$

So $x > 100$

$0 < b < 1$

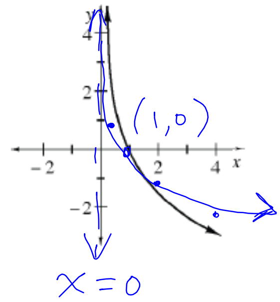
6-98. At right is a graph of $y = \log_b x$. Describe the possible values for b.

Check: $y = \log_{\frac{1}{2}} x$

$\left(\frac{1}{2}\right)^y = x$

2^2

x	y
1	0
$\frac{1}{2}$	1
2	-1
4	-2



6-99. Consider the questions below.

a. What can you multiply 8 by to get 1?

b. What can you multiply x by to get 1?

c. Using the rules of exponents, find a way to solve $m^8 = 40$. Remember that logarithms will not be useful here, but the exponent key on your calculator *will* be. (Obtain the answer as a decimal approximation using your calculator. Check your result by raising it to the 8th power.)

d. Now solve $n^6 = 300$.

e. Describe a method for solving $x^a = b$ for x with a calculator.

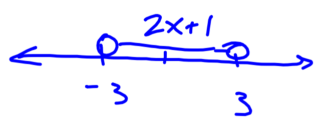
6-100. Adam keeps getting negative exponents and fractional exponents confused. Help him by explaining the difference between $2^{1/2}$ and 2^{-1}.

$\sqrt{2}$ $\qquad$ $\frac{1}{2}$

6-101. Solve each inequality and graph its solution on a number line.

a. $|x| < 3$ $\qquad$ b. $|2x+1| < 3$ $\qquad$ c. $|2x+1| \geq 3$

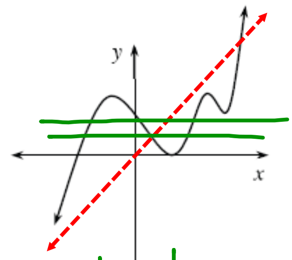
$-3 < 2x+1 < 3$



Not connected.

Need 2 separate inequalities

6-102. Consider the graph at right.



a. Is the graph a function? Explain.

b. Make a sketch of the inverse of this graph. Is the inverse a function? Justify your answer.

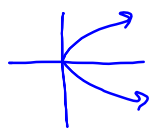
c. Must the inverse of a function be a function? Explain.

d. Describe what is characteristic about functions that do have inverse functions. → Look at horiz. line test on original

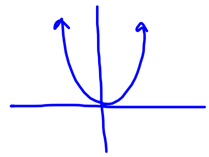
e. Could the inverse of a non-function be a function? Explain or give an example.

yes!

non-function:



inverse:



6-103. Solve each system of equations below.

a. $-4x = z - 2y + 12$
$y + z = 12 - x$
$8x - 3y + 4z = 1$

b. $3x + y - 2z = 6$
$x + 2y + z = 7$
$\frac{6x}{2} + \frac{2y}{2} - \frac{4z}{2} = \frac{12}{2}$

c. What does the solution in part (b) tell you about the graphs?

Since 2 of the equations represent the same plane, the other plane will intersect it in a line. (The other one is not parallel to it.)

Rewrite

① $-4x + 2y - z = 12$

② $(x + y + z = 12)(-4)$

③ $8x - 3y + 4z = 1$

Plan: Eliminate z

① + ② → $-3x + 3y = 24$
$-x + y = 8$

③ + ②(-4) →

$-4x - 4y - 4z = -48$
$8x - 3y + 4z = 1$

Wednesday's CP's:

6-68. THE COMMUTER

Sensible Sally has a job that is 35 miles from her home and needs to be at work by 8:15 a.m. She wants to get as much sleep as she can, leave as late as possible, and still get to work on time. Sally discovered that if she leaves at 7:10, it takes her 40 minutes to get to work. If she leaves at 7:30, it takes her 60 minutes to get work. If she leaves at 7:40, it takes her 50 minutes to get to work. Since her commute time increases and then decreases, Sally decided to use a parabola to model her commute, assuming the time it takes her to get to work varies quadratically with the number of minutes after 7:00 that she leaves her house.

x y

(10,40)
(30,60)
(40,50)

a. If x = the number of minutes after 7:00 that Sally leaves, and y = the number of minutes it takes Sally to get to work, what three ordered pairs can you determine from the problem?

b. Use the three points from part (a) to find the equation of a parabola in standard form that can be used to model Sally's commute.

c. Will Sally make it to work on time if she leaves at 7:20?

$y = ax^2 + bx + c$

$(10,40) \longrightarrow 40 = a(10)^2 + b(10) + c$

① $100a + 10b + c = 40$

$(30,60) \longrightarrow$ ②

$(40,50) \longrightarrow$ ③

$y = -\frac{1}{15}x^2 + \frac{11}{3}x + 10$

6-89. Marta was convinced that there had to be some way to graph $y = \log_2 x$ on her graphing calculator. She typed in $y = \log(2^x)$ and pressed GRAPH.

"It WORKED!" Marta yelled in triumph.

"Whaaaat?" said Celeste. *"I think $y = \log_2 x$ and $y = \log(2^x)$ are totally different, and I bet we can show it by converting both of them to exponential form."*

"Yeah, I think you're wrong, Marta," said Sophia. *"I think we can show that $y = \log_2 x$ and $y = \log(2^x)$ are totally different by looking at the graphs."*

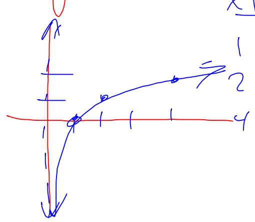
a. Show that the two equations are different by sketching the graph of $y = \log_2 x$. Then sketch what your graphing calculator shows to be the graph of $y = \log(2^x)$.

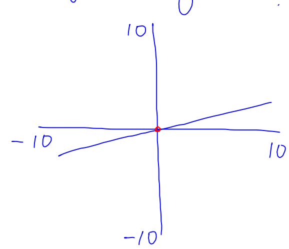
b. Now show that $y = \log_2 x$ and $y = \log(2^x)$ are different by converting both of them to exponential form.

$y = \log_2 x \rightarrow 2^y = x$

$y = \log 2^x$

x	y
1	0
2	1
4	2





$2^y = x$

$10^y = 2^x$

Yesterday's CP's:

6-90. The work you did in problem 6-89 is a **counterexample**, which shows that in general, the statement $\log_b x = \log(b^x)$ is *false*. For each of the following log statements, use the strategies from problem 6-89 to determine whether they are true or false, and justify your answer. Be ready to present your conclusions and justifications.

a. $\log_5(25) \overset{?}{\neq} \log_{25}(5)$

b. $\log(x^2) \overset{?}{\neq} (\log x)^2$

c. $\log(7^x) \overset{?}{=} x\log(7)$

d. $\log(2x) \overset{?}{\neq} \log_2 x$

6-91. In the previous problem only *one* of the statements was true.

a. Use different numbers to make up four more statements that follow the same pattern as the one true statement, and test each one to see whether it appears to be true.

b. Use your results to complete the following statement, which is known as the **Power Property of Logarithms**: $\log(b^x) =$ $x(\log b)$

$y = \log b^x$

c) $\log 7^x = x(\log 7)$

let $b = 2$, let $x = 3$ $\rightarrow \log 2^3 \overset{?}{=} 3(\log 2)$

≈ 0.903 ≈ 0.903

let $b = 5$, $x = 4$ $\rightarrow \log 5^4 \overset{?}{=} 4(\log 5)$

≈ 2.796 ≈ 2.796

let $b =$

6-93. THERE MUST BE AN EASIER WAY

It would certainly be helpful to have an easier method than guess and check to solve equations like $1.04^x = 2$. Complete parts (a) through (c) below to discover an easier way.

a. What makes the equation $1.04^x = 2$ so hard to solve?

b. Surprise! In the first part of this lesson, you already found a method for getting rid of inconvenient exponents! Talk with your team about how your results from problems 6-90 and 6-91 can help you rewrite the equation $1.04^x = 2$. Be prepared to share your ideas with the class.

c. Solve $1.04^x = 2$ using this new method. Be sure to check your answer.

a) $1.04^x = 2$ ① variable is an exponent ② Can't make the bases match

New Method: Take the log of both sides

b) $\log 1.04^x = \log 2$

$$\frac{x(\cancel{\log 1.04})}{(\cancel{\log 1.04})} = \frac{\log 2}{\log 1.04}$$

$x \approx 17.67$

check

$1.04^{17.67} \overset{?}{\approx} 2$ ✓

CP's: 6 - #104 ---> 106

6.2.2 How can I rewrite it?

Investigating the Properties of Logarithms

You already know the basic rules for working with exponents. Since logs are the inverses of exponential functions, they also have properties that are similar to the ones you already know. In this lesson, you will explore these properties.

6-104. Marta now knows that if she wants to find $\log_2(30)$, she cannot just type $\log(2^{30})$ into her calculator, since her calculator's log key cannot directly calculate logs with base 2. But she still wants to be able to find what $\log_2(30)$ equals.

a. First, use your knowledge of logs and exponents to estimate $\log_2(30)$.

b. Now use what you learned in Lesson 6.2.1 to get a better estimate. Since you want to determine what $\log_2(30)$ equals, you can write $\log_2(30) = x$. When working with a log equation, it is often easier to first convert it to exponential form. Rewrite this equation in exponential form.

c. Use the methods you developed in class to solve this equation. Refer back to your work on problem 6-93 if you need help.

6-105. Congratulations! You are smarter than your calculator. You have just evaluated a log with base 2, even though your calculator does not do that. Now you will practice some more.

a. First estimate an answer, then apply the method you have just developed to evaluate $\log_5(200)$.

b. Apply the process you used in part (a) to evaluate the expression $\log_a b = x$.

5^3 125 5^4 625

$5^? = 200$

$? \approx 3.25$

3.3

$\log_5 200 = x$

$5^x = 200$

$\log 5^x = \log 200$

$\frac{x(\log 5)}{\log 5} = \frac{\log 200}{\log 5}$

$x \approx 3.29$

b) $\log_a b = x$

$a^x = b$

$\log a^x = \log b$

$\frac{x(\cancel{\log a})}{\cancel{\log a}} = \frac{\log b}{\log a}$

x x

$\log_a b = \frac{\log_{10} b}{\log_{10} a}$

6-106. Since logs and exponentials are inverses, the properties of exponents (which you already know) also apply to logs. The problems below will help you discover these new log properties.

a. Complete the two exponent rules below. In part (b), you will find the equivalent properties for logs.

$x^a x^b = x^{a+b}$ and $\frac{x^b}{x^a} = x^{b-a}$

b. To help you find the equivalent log properties, use your calculator to solve for x in each problem below. Note that x is a whole number in parts (*i*) through (*v*). Look for patterns that would make your job easier and allow you to generalize in part (*vi*).

i. $\log(5)+\log(6)=\log(x)$ *ii.* $\log(5)+\log(2)=\log(x)$

iii. $\log(5)+\log(5)=\log(x)$ *iv.* $\log(10)+\log(100)=\log(x)$

v. $\log(9)+\log(11)=\log(x)$ *vi.* $\log(a)+\log(b)=\log(\_\_\_\_)$

c. What if the log expressions are being subtracted instead of added? Solve for x in each problem below. Note that x will not always be a whole number. Again, look for patterns that will allow you to generalize in part (*vi*).

i. $\log(20)-\log(5)=\log(x)$ *ii.* $\log(30)-\log(3)=\log(x)$

iii. $\log(5)-\log(2)=\log(x)$ *iv.* $\log(17)-\log(9)=\log(x)$

v. $\log(375)-\log(17)=\log(x)$ *vi.* $\log(b)-\log(a)=\log(\_\_\_\_)$

Week 5 Classwork:

Warm Up on top

6- #30 ---> 33, 35

(dot paper: #31, 32, 35)

6- #44 ---> 48

(dot paper: #45)

6- #60, 61, 64, 65

6- #66 ---> 68, 70

6- #88 --->93

HW: 6-

113 ---> 118